and the area of that part of the cone lying above a region D in plane and within the unit circle is

$$\begin{aligned} A &= \iint_D \sqrt{1+(f_u)^2+(f_v)^2}\,du\,dv \\ &= \iint_D \sqrt{1+\frac{a^2u^2}{u^2+v^2}+\frac{a^2v^2}{u^2+v^2}}\,du\,dv \\ &= \iint_D \sqrt{1+a^2}\,du\,dv \;=\; \text{Area}(D)\sqrt{a^2+1}\,, \end{aligned}$$

which yields the desired conclusion.

Also solved by OLIVER GEUPEL, Brühl, NRW, Germany; RICHARD I. HESS, Rancho Palos Verdes, CA, USA; WALTHER JANOUS, Ursulinengymnasium, Innsbruck, Austria; MISSOURI STATE UNIVERSITY PROBLEM SOLVING GROUP, Springfield, MO, USA; and the proposers.

3443. [2009 : 234, 236] *Proposed by Cao Minh Quang, Nguyen Binh Khiem High School, Vinh Long, Vietnam.*

Let a, b, and c be positive real numbers such that $a+b+c=3$. Prove that

$$\sum_{\text{cyclic}} \frac{a^2(b+1)}{a+b+ab} \;\geq\; 2\,.$$

Solution by Arkady Alt, San Jose, CA, USA.

We have

$$\begin{aligned} \sum_{\text{cyclic}} \frac{a^2(b+1)}{a+b+ab} &= \sum_{\text{cyclic}} \left(\frac{a^2(b+1)}{a+b+ab} - a + 1\right) \\ &= \sum_{\text{cyclic}} \frac{a+b}{a+b+ab} \;\geq\; \sum_{\text{cyclic}} \frac{a+b}{a+b+\frac{(a+b)^2}{4}} \\ &= \sum_{\text{cyclic}} \frac{4}{4+a+b} \;=\; \frac{4}{18}\cdot\sum_{\text{cyclic}}(4+a+b)\sum_{\text{cyclic}}\frac{1}{4+a+b} \\ &\geq \frac{4}{18}\cdot 9 \;=\; 2\,, \end{aligned}$$

where we used the fact that $(x+y+z)(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}) \geq 9$ for positive real numbers x, y, z and that $\sum\limits_{\text{cyclic}}(4+a+b) = 18$.

Also solved by GEORGE APOSTOLOPOULOS, Messolonghi, Greece; ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; OLIVER GEUPEL, Brühl, NRW, Germany; JOHN G. HEUVER, Grande Prairie, AB; JOE HOWARD, Portales, NM, USA; HUNEDOARA PROBLEM SOLVING GROUP, Hunedoara,